Time-dependent evolution of two coupled Luttinger liquids

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We consider two disconnected Luttinger liquids which are coupled at $t = 0$ through chiral density-density interactions. Both for $t < 0$ and $t \geq 0$ the system is exactly solvable by means of bosonization and this allows to evaluate analytically the time-dependence of correlation functions. We find that in the long-time limit the critical exponent governing the one-particle correlation function differs from the exponent dictated by the equilibrium ground state of the coupled system. We also discuss how this reflects on some physical quantities which are accessible to real experiments.

Time-dependent quantum systems are attracting much the attention during the last years. This is because the dynamical response of nanoscopic devices is nowadays accessible in many experiments with high relevance in practical applications such as quantum computing and single-electron transport[1]. On one side we mention the wide investigation of out-of-equilibrium phenomena in transport experiments, where time-dependent transient currents are measured at picosecond time-scales[2]. On the other side, recent experiments in ultracold atoms confined in optical lattices have shown that it is possible to time-tune the strength of the interactions in both bosonic[3] and fermionic[4] systems, by using the so called Feshbach resonance. In these cases it arises the intriguing question of what is the steady state to which the initial ground state relaxes to[5][6].

In the light of this challenging physics, the theoretical investigation of the time-dependent evolution of many-body interacting systems deserves special attention. Recently, an exact formulation of time-dependent transport in electronic systems[7] was derived in the framework of time-dependent-density-functional-theory[8]. Unfortunately its implementation to strongly interacting systems results hard. An efficient numerical tool to address the problem of the electron-electron (e-e) interactions is the time-dependent-density-matrix-renormalization-group[9][10][11], which turns out to have excellent performance in one-dimensional (1D) systems. For what concerns electronic systems with time-dependent parameters but not involving charge transport, in a recent paper Cazalilla carried out the exact evolution of a noninteracting 1D system following a sudden switch-on of Luttinger liquid e-e interaction[12]. Remarkably, it is found that the initial ground state reaches a stationary state only if the system is infinite-sized and that its asymptotic behavior differs from the one of the interacting equilibrium system[6].

In the present paper we extend this analysis to two coupled interacting 1D systems. Our system consists of two isolated Luttinger liquids which are connected at $t =$ 0 through interactions between chiral electron densities located at different liquids.

The Luttinger liquid is the prototype of interacting electrons in 1D and it is governed by the so called Tomonaga-Luttinger Hamiltonian. In this model, the electrons have linear dispersion relation around positive (Right) and negative (Left) Fermi points located at $\pm k_F$ and the e-e interactions act only between Right/Left electron densities. This means that only the component with momentum transfer $p \sim 0$ of the Coulomb repulsion is retained, while the $p \sim 2k_F$ component is assumed to be negligible. The model is exactly solvable by means of bosonization technique, which allows to write the electron Hamiltonian in terms of boson operators b's.

In the following we shall adopt a similar notation as in Ref. 12.

For $t < 0$ the system consists of two disconnected identical interacting electron liquids described by the bosonized Hamiltonian[13] $H_0 = H_1 + H_2$ with

$$H_i = \frac{1}{2}\sum_{q\neq 0} \quad [(v_F + g_4^{(4)}(q))|q|(b_i^\dagger(q)b_i(q) + b_i(q)b_i^\dagger(q))$$
$$-g_4^{(2)}(q)(b_i^\dagger(q)b_i^\dagger(-q) + b_i(q)b_i(-q))] \quad (1)$$

where $i = 1, 2$, $[b_i(q), b_j^\dagger(q')] = \delta_{i,j}\delta_{q,q'}$, v_F is the Fermi velocity, $g_4^{(4)}(q)$ is the interaction parameter between Right-Right (positive q) and Left-Left (negative q) electron densities, while $g_4^{(2)}(q)$ is the interaction parameter between Left-Right densities.

The system is diagonalized by the well-known Bogoliubov transformation:

$$\tilde{b}_i(q) = \cosh\varphi(q)b_i(q) + \sinh\varphi(q)b_i^\dagger(-q),$$
$$\tilde{b}_i^\dagger(q) = \sinh\varphi(q)b_i(-q) + \cosh\varphi(q)b_i^\dagger(q) \quad (2)$$

with

$$\tanh 2\varphi(q) = g_4^{(2)}(q)/[v_F + g_4^{(4)}(q)], \quad (3)$$

and renormalized velocity

$$v(q) = \sqrt{(v_F + g_4^{(4)}(q))^2 - (g_4^{(2)}(q))^2}. \quad (4)$$

For $t \geq 0$ the chiral density-density interactions between the two Luttinger liquids are switched on and the system is governed by the total Hamiltonian

$$H_{tot} = H_0 + \theta(t)H_{12} \qquad (5)$$

where

$$
\begin{aligned}
H_{12} = & \sum_{q \neq 0} [g_2^{(4)}(q)(b_1^\dagger(q)b_2(q) + b_2^\dagger(q)b_1(q) \\
& + b_1(q)b_2^\dagger(q) + b_2(q)b_1^\dagger(q) \\
& - g_2^{(2)}(q)(b_1^\dagger(q)b_2^\dagger(-q) + b_1(q)b_2(-q) \\
& + b_2^\dagger(q)b_1^\dagger(-q) + b_2(q)b_1(-q))] ,
\end{aligned}
\qquad (6)
$$

where $g_2^{(4)}(q)$ ($g_2^{(2)}(q)$) is the interaction parameter between electron densities of the same (opposite) chirality in different liquids. Physically H_{12} could represent the long-range component of the Coulomb repulsion felt by electrons located in two (quasi)1D metallic systems close to each other. Here we are considering a general situation where $g_2^{(4)}(q) \neq g_2^{(2)}(q)$ but in real systems like carbon nanotubes, they coincide (see below). It is worth to remark that in the present model the coupling between the two liquids does not involve any inter-liquid electron tunneling.

H_{tot} is again diagonalized by introducing symmetrized and antisymmetrized operators[14]:

$$
\begin{aligned}
s(q) &= [b_1(q) + b_2(q)]/\sqrt{2} , \\
a(q) &= [b_1(q) - b_2(q)]/\sqrt{2} .
\end{aligned}
\qquad (7)
$$

In terms of them H_{tot} decouples in two independent non-equivalent Luttinger liquids:

$$
\begin{aligned}
H_{tot} &= H_s + H_a \\
&= \frac{1}{2}\sum_{q \neq 0} [(v_F + g_4^{(4)}(q) + g_2^{(4)})|q|(s^\dagger(q)s(q) + s(q)s^\dagger(q)) \\
&\quad - (g_4^{(2)}(q) + g_2^{(2)}(q))(s^\dagger(q)s^\dagger(-q) + s(q)s(-q))] \\
&\quad + \frac{1}{2}\sum_{q \neq 0}[(v_F + g_4^{(4)}(q) - g_2^{(4)})|q|(a^\dagger(q)a(q) + a(q)a^\dagger(q)) \\
&\quad - (g_4^{(2)}(q) - g_2^{(2)}(q))(a^\dagger(q)a^\dagger(-q) + a(q)a(-q))] .
\end{aligned}
\qquad (8)
$$

Finally, the diagonal form of H_{tot} is obtained in terms of

$$
\begin{aligned}
\tilde{s}(q) &= \cosh\varphi_s(q)s(q) + \sinh\varphi_s(q)s^\dagger(-q) , \\
\tilde{s}^\dagger(q) &= \sinh\varphi_s(q)s(-q) + \cosh\varphi_s(q)s^\dagger(q) , \\
\tilde{a}(q) &= \cosh\varphi_a(q)a(q) + \sinh\varphi_a(q)a^\dagger(-q) , \\
\tilde{a}^\dagger(q) &= \sinh\varphi_a(q)a(-q) + \cosh\varphi_a(q)a^\dagger(q) ,
\end{aligned}
\qquad (9)
$$

where

$$\tanh 2\varphi_{s,a}(q) = [g_4^{(2)}(q) \pm g_2^{(2)}(q)]/[v_F + g_4^{(4)}(q) \pm g_2^{(4)}(q)] , \qquad (10)$$

and renormalized velocities

$$v_{s,a}(q) = \sqrt{(v_F + g_4^{(4)}(q) \pm g_2^{(4)}(q))^2 - (g_2^{(2)}(q) \pm g_4^{(2)}(q))^2} . \qquad (11)$$

Now we are in the position to evaluate the equal-time one-particle correlation function for $t > 0$, defined as

$$G_\gamma^{(i)}(x,t) = \langle e^{iH_{tot}t}\psi_{i,\gamma}(x)\psi_{i,\gamma}^\dagger(0)e^{-iH_{tot}t}\rangle_{H_0} \qquad (12)$$

where $i = 1, 2$ labels the two Luttinger liquids decoupled at $t < 0$, the subscript $\gamma = R, L$ indicates the Right/Left character of the non-interacting fermion fields ψ and $\psi^\dagger$, and $\langle \ldots \rangle_{H_0}$ is the zero-temperature average in the ensemble H_0. Without any loss of generality we shall focus on $G_R^{(1)}(x,t)$.

The bosonization technique allows to calculate G_R with logarithmic accurancy, which is exact for distances/times much longer than the typical range of the interactions. The key point of bosonization is that it is possible to express the fermion field in terms of boson fields. For instance the Right-mover fermion field is given by the following expression:

$$\psi_{i,R}(x) = \frac{\eta_R}{(2\pi\alpha)^{1/2}}e^{i\Phi_{i,R}(x)} , \qquad (13)$$

where α is a short-distance cutoff, proportional to the lattice spacing, η_R is an anticommuting Klein factor and

$$
\begin{aligned}
\Phi_{i,R}(x) = & \sum_{q>0}\left(\frac{2\pi}{qL}\right)^{1/2} e^{-\alpha q/2}[b_i^\dagger(q)e^{-iqx} + b_i(q)e^{iqx}] \\
& + \varphi_{0,R} + 2\pi x N_R/L ,
\end{aligned}
\qquad (14)
$$

where N_R is the total number of Right-electrons, $[\varphi_{0,R}, N_R] = i$ and L is the length of the system.

Thus the computation of the correlation function reduces to:

$$G_R^{(1)}(x,t) = \frac{1}{2\pi\alpha}\langle e^{i\Phi_{1,R}(x,t)}e^{-i\Phi_{1,R}(0,t)}\rangle_{H_0} =$$

$$\frac{1}{2\pi\alpha}e^{-\frac{1}{2}\{\langle(\Phi_{1,R}(x,t)-\Phi_{1,R}(x,t))^2\rangle_{H_0} - [\Phi_{1,R}(x,t),\Phi_{1,R}(0,t)]\}} \qquad (15)$$

where $\Phi_{1,R}(x,t) = e^{iH_{tot}t}\Phi_{1,R}(x)e^{-iH_{tot}t}$. In order to compute $\Phi_{1,R}(x,t)$ it is convenient to evaluate first $b_1(q,t) = e^{iH_{tot}t}b_1(q)e^{-iH_{tot}t}$ in terms of the $\tilde{b}_i(q)$'s and $\tilde{b}_i^\dagger(q)$'s, since they diagonalize H_0 which defines the ensemble we average on.

After a lengthy algebra involving a direct and an inverse Bogolioubov transformation in Eq. 9 and the inverse of Eq. 2, one finds

$$
\begin{aligned}
b_1(q,t) = & A(q,t)\tilde{b}_1(q) + B^\star(q,t)\tilde{b}_1^\dagger(-q) \\
& + C(q,t)\tilde{b}_2(q) + D^\star(q,t)\tilde{b}_2^\dagger(-q) ,
\end{aligned}
\qquad (16)
$$

where

$$
\begin{aligned}
A(q,t) &= \cosh\varphi(q)\left[-i\sin v_s|q|t\cosh 2\varphi_s(q)\right.\\
&\quad \left.+\cos v_s|q|t+(a\leftrightarrow s)\right]\\
&\quad -\sinh\varphi(q)\left[-i\sin v_s|q|t\sinh 2\varphi_s(q)\right.\\
&\quad \left.+(a\leftrightarrow s)\right],\\
B^\star(q,t) &= -\sinh\varphi(q)\left[-i\sin v_s|q|t\cosh 2\varphi_s(q)\right.\\
&\quad \left.+\cos v_s|q|t+(a\leftrightarrow s)\right]\\
&\quad +\cosh\varphi(q)\left[-i\sin v_s|q|t\sinh 2\varphi_s(q)\right.\\
&\quad \left.+(a\leftrightarrow s)\right],\\
C(q,t) &= \cosh\varphi(q)\left[-i\sin v_s|q|t\cosh 2\varphi_s(q)\right.\\
&\quad \left.+\cos v_s|q|t-(a\leftrightarrow s)\right]\\
&\quad -\sinh\varphi(q)\left[-i\sin v_s|q|t\sinh 2\varphi_s(q)\right.\\
&\quad \left.-(a\leftrightarrow s)\right],\\
D^\star(q,t) &= -\sinh\varphi(q)\left[-i\sin v_s|q|t\cosh 2\varphi_s(q)\right.\\
&\quad \left.+\cos v_s|q|t-(a\leftrightarrow s)\right]\\
&\quad +\cosh\varphi(q)\left[-i\sin v_s|q|t\sinh 2\varphi_s(q)\right.\\
&\quad \left.-(a\leftrightarrow s)\right].
\end{aligned}
\tag{17}
$$

The above relations allow to evaluate exactly the time-dependence in the correlation function, which reads:

$$
G_R^{(1)}(x,t) = \frac{1}{2\pi\alpha}\exp[-\sum_{q>0}\left(\frac{2\pi}{qL}\right)e^{-\alpha q}
$$
$$
\times[(|A(q,t)|^2+|C(q,t)|^2)(-i\sin qx+1-\cos qx)
$$
$$
+(|B(q,t)|^2+|D(q,t)|^2)(i\sin qx+1-\cos qx)]]\tag{18}
$$

The sum over q is performed by using that $q=2\pi n/L$ and we end up with

$$
G_R^{(1)}(x,t) = \frac{c}{d(x)^{\frac{1}{4}(2\cosh 2\varphi+\cosh(2\varphi-4\varphi_s)+\cosh(2\varphi-4\varphi_a))}}
$$
$$
\times\left[\frac{d(x-2v_st)d(x+2v_st)}{d(2v_st)^2}\right]^{\frac{1}{8}(\cosh(2\varphi-4\varphi_s)-\cosh 2\varphi)}
$$
$$
\times\left[\frac{d(x-2v_at)d(x+2v_at)}{d(2v_at)^2}\right]^{\frac{1}{8}(\cosh(2\varphi-4\varphi_a)-\cosh 2\varphi)}
\tag{19}
$$

where $d(x)=L\sin(\pi x/L)$. In order to obtain the above expression we have introduced momentum-independent φ and $\varphi_{s,a}$ such that $\sinh f(q)\approx e^{-R_0|q|/2}\sinh f(0)$ with R_0 of the order of the range of the e-e interactions; in this way the finite overall prefactor c is produced[15].

Let us now consider some relevant limits.

First, it is straightforward to check that in the non-interacting case (i.e. $\varphi=\varphi_s=\varphi_a=0$ and $v_a=v_s=v_F$) we recover

$$
G_R^{(1)}(x,t) = \frac{c}{d(x)};
\tag{20}
$$

on the other hand for $t\to 0$ we find

$$
G_R^{(1)}(x,0) = \frac{c'}{d(x)^{\cosh 2\varphi}},
\tag{21}
$$

where c' is again a finite constant; this result corresponds to the correlation function of an individual Luttinger liquid governed by H_1 or H_2, as it should.

More interesting is the case in which $g_4^{(2)}=g_4^{(4)}=g_2^{(4)}=0$ (i.e. $\varphi=0$ and $-\varphi_a=\varphi_s\equiv\bar\varphi\neq 0$ and $v_a=v_s\equiv\bar v\neq v_F$). In this case the system corresponds to have we two identical replicas of the model considered by Cazalilla, i.e. an isolated noninteracting 1D system with density-density interactions switched on at $t=0$. In this case the we obtain

$$
G_R^{(1)}(x,t) = \frac{c}{d(x)^{\frac{1}{4}(\cosh 4\bar\varphi+1)}}
$$
$$
\times\left[\frac{d(x-2\bar vt)d(x+2\bar vt)}{d(2\bar vt)^2}\right]^{\frac{1}{4}(\cosh 4\bar\varphi-1)}
\tag{22}
$$

which recovers the result of Cazalilla (Eq. 9 of Ref. 12).

Anyway the most remarkable result is obtained from the long-time limit $t\to\infty$ of Eq. 19. In this case we get

$$
G_R^{(1)}(x) \approx \frac{c}{|x|^{\frac{1}{4}(2\cosh 2\varphi+\cosh(2\varphi-4\varphi_s)+\cosh(2\varphi-4\varphi_a))}}
\tag{23}
$$

where we have also taken $L\to\infty$. We point out that if the infinite length limit is not taken, the correlations obey an oscillatory behavior even at long time, confirming that in a finite-sized system a steady state is not reachable. However, as already discussed in Ref. 12, in mesoscopic systems with finite L (e.g. $L\sim 1\mu$m for carbon nanotubes) a genuine stationary state can be reached because of finite temperature effects. Indeed if the temperature T is larger than the level spacing $\Delta\varepsilon\sim v_F/L$, we have to replace L by v_F/T and sin by sinh in the definition of $d(x)$. As a consequence for very large distances/times the Green function decays exponentially as $G_R^1\sim e^{-|x|\nu T}$, where ν is the same exponent displayed in the power-law at $T=0$[16]. Thus it appears that in the exponential regime we can still keep track of the Luttinger liquid correlations encoded by ν. On the other hand for distances/times smaller than the thermal length v_F'/T (with v_F' the renormalized Fermi velocity[16]) the power-law discussed above still holds.

In the thermodynamic limit, it is worth to notice that the result in Eq. 23 does not coincide with the equilibrium equal-time correlation function of a two coupled Luttinger liquids governed by H_0+H_{12}. Indeed in the latter case the correlation function reads

$$
G_R^{(1)}(x) \approx \frac{c''}{|x|^{\frac{1}{2}(\cosh(2\varphi_s)+\cosh(2\varphi_a))}},
\tag{24}
$$

with finite c''. The difference in the critical exponents in the two cases reveals that in the long-time limit the ground state of H_0 evolved by $H_0+\theta(t)H_{12}$ reaches a stationary state which is *not* the ground state of H_0+H_{12}. This is due to the fact that for $t>0$ the energy is

conserved and therefore the system cannot relax to the ground state. Indeed the critical exponent in Eq. 23 is larger than the critical exponent in Eq. 24, consistently with the fact that in the latter case the system is able to optimize the repulsive interactions given by H_{12}.

Now let us discuss physical consequences of this result. To this end we introduce the critical exponent α governing some relevant observables like the tunneling density of states ρ[17]

$$\rho(\omega) \sim \omega^{\alpha} \qquad (25)$$

which is detectable with scanning tunneling microscopy by probing the bulk the 1D conductor[18][19].

In order to compare to real systems, we have to introduce spin in the previous analysis. This is completely strightforward as long as we consider spin-independent e-e interactions. For spinless electrons, the α exponent is obtained by subtracting 1 to the exponent one-particle correlation function, while for the corresponding spinful system we just have to half such a value[25].

Anyway it is worth to note that, while in equilibrium systems the critical exponent of $\rho(\omega)$ can be safely extracted from the equal-time one-particle Green function, in out-of-equilibrium situations such a procedure is far from obvious[20]. Therefore it would be convenient to relate α to some other energy-independent observable, like the momentum distribution, which also displays a power law[17]:

$$n(q) \propto \text{sign}(q - k_F)|q - k_F|^{\alpha} . \qquad (26)$$

$n(q)$ is accessible, for instance, from angle-resolved-photo-emission-spectroscpy, which has been successfully applied to quasi-1D meterials[21][22] to measure α[23][24]. We also mention that such a technique is not affected by the uncertainties regarding the contacts, which are present in transport experiments. At this point, we note again that the above power-law would be cutoff by finite temperature effects if $|q - k_F| \sim T/v_F$.

From Eqs. 23 and 24 the exponent governing $n(q)$ in the two different cases discussed above read

$$\alpha_{\text{asympt}} =$$
$$\frac{1}{8}(2\cosh 2\varphi + \cosh(2\varphi - 4\varphi_s) + \cosh(2\varphi - 4\varphi_a)) - \frac{1}{2},$$
$$\alpha_{\text{equil}} = \frac{1}{4}(\cosh(2\varphi_s) + \cosh(2\varphi_a)) - \frac{1}{2}. \qquad (27)$$

The model we have considered could find an experimental realization in 1D fermionic systems built within optical potential. Unfortunately in these systems the interaction is short-ranged and this makes the term H_{12} hard to realize. Anyway long-range dipole-dipole interactions have been recently obtained in chromium bosonic atoms[26] and Fermi gases with long-range interactions are likely becoming available.

On the other hand it is tempting to relate our results to the physics of carbon nanotubes. Metallic carbon nanotubes are believed to be rather good (although approximate) realizations of Luttinger liquids since in normal conditions the main correlation effects come from the long-range part of the Coulomb repulsion (through the $g_i^{(j)}$ parametres), while the back-scattering interactions with large momentum transfer suffer a $1/R$ suppression, where R is the radius of the nanotube[18][19]. Indeed back-scattering interactions are marginal operators in the renormalization group sense and in carbon nanotubes are predicted to breakdown the Luttinger liquid state only at very low energy scales[18]. This is supported by direct observation of power-law decay of the linear conductance as a function of temperature[27][28][29].

It is worth to recall that each individual carbon nanotube is composed itself by two coupled (identical) Luttinger liquids, since there are a Left and a Right branch respectively around each of the two Fermi points[14][18][30]. This implies that the critical exponent governing the decay of the correlation functions in these systems is actually one half the exponents defined in Eqs. 27. Therefore in the following we will use $\alpha_{\text{asympt}}^{\text{(nano)}} = \alpha_{\text{asympt}}/2$ and $\alpha_{\text{equil}}^{\text{(nano)}} = \alpha_{\text{equil}}/2$.

The idea is to compare two possible measurements realized in two different experimental setups. The first system (setup A), which gives access to $\alpha_{\text{asympt}}^{\text{(nano)}}$, is made by two (initially distant) nanotubes which can be quickly aligned at controlled distance. This makes possible to mimic the switching on of inter-tube interactions. In this case, of course, the critical exponent could be only be measured in the final steady state and any transient regime is definitively unaccessible. The second system (setup B), which gives access to $\alpha_{\text{equil}}^{\text{(nano)}}$, consists in having the carbon nanotubes synthetized at a given distance, for instance in a double-wall nanotube. In this case the system has fixed inter-tube interactions built-in from the very beginning.

Typical metallic nanotubes correspond to armchair $(10, 10)$ geometry (i.e. radius $R \approx 7\text{Å}$) with length $L \sim 1\mu\text{m}$. The strength of the density-density Coulomb interactions is then[18][30]

$$g_4^{(2)}(q \sim 0) = g_4^{(4)}(q \sim 0) \approx \frac{4e^2}{\kappa \pi v_F} K_0(q_c R) ,$$
$$g_2^{(2)}(q \sim 0) = g_2^{(4)}(q \sim 0) \approx \frac{4e^2}{\kappa \pi v_F} K_0(q_c D) ,$$
$$(28)$$

where K_0 is the modified Bessel function, $q_c = 2\pi/L$ is the infrared momentum cutoff due to the finite size of the nanotubes, $D \gtrsim R$ is the intertube distance, $\kappa \sim 2$ is the dielectric constant of typical nanotubes and $v_F \approx 8 \times 10^5$m/s. We recall at this point that the amplitude of intra-tube backscattering interactions is approximatively[18] $0.1e^2 a/\kappa \pi v_F R$ (with $a = 2.46\text{Å}$)
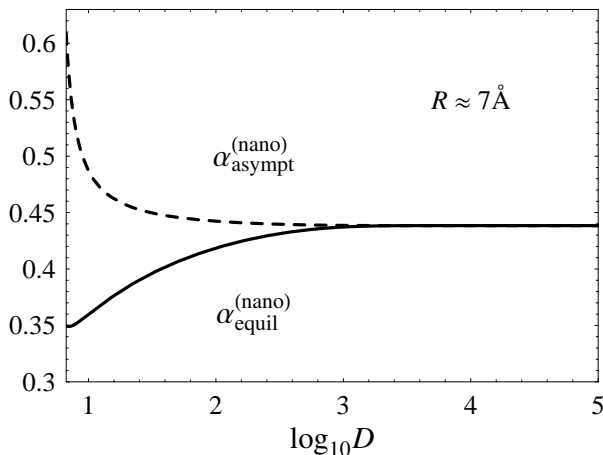
FIG. 1: Plot of the critical exponents $\alpha_{\text{asympt}}^{(\text{nano})}$ (dashed curve) and $\alpha_{\text{equil}}^{(\text{nano})}$ (solid curve) for typical $(10, 10)$ nanotubes as a function of the logarithm of the distance D between the nanotubes. In $\log_{10} D$, D is expressed in Å.

which is much smaller of both intra- and inter-tube forward scattering interactions for $D \gtrsim R$. We also mention that single-electron tunneling between nanotubes is usually strongly suppressed since the misalignment of the respective carbon lattices prevents momentum conservation in the tunneling process. Therfore the experimental setups that we propose could be fairly approximated by the model of two coupled Luttinger liquids.

By inserting $g_i^{(j)}$ obtained in Eqs. 28 in Eqs. 3 and 10, we can estimate the critical exponents accessible to experiments within setups A and B.

The two exponents are plotted in Fig. 1 as a function of the distance between the nanotubes. For $D \to \infty$ the intertube couplings $g_2^{(4)}$ and $g_2^{(2)}$ vanish (see Eq. 28) and $\alpha_{\text{asympt}}^{(\text{nano})}$ and $\alpha_{\text{equil}}^{(\text{nano})}$ tend to the common value

$$\alpha_{\text{disc}}^{(\text{nano})} = \frac{1}{4}(\cosh 2\varphi - 1) \approx 0.43\,, \qquad (29)$$

which governs the correlations of two completely independent nanotubes, according to Eq. 21. This value is in agreement with previous estimates given for single typical nanotubes[18][19].

When the two nanotubes are brought close to each other in setup A (Fig. 1 dashed line), the intertube interaction are switched on and $\alpha_{\text{asympt}}^{(\text{nano})}$ increases. In particular at $D \sim R$ we find

$$\alpha_{\text{asympt}}^{(\text{nano})} \approx 0.60\,. \qquad (30)$$

Regarding setup B (Fig. 1 solid line), if the two carbon nanotubes have been synthetized at the same distance $D \sim R$, the predicted exponent is

$$\alpha_{\text{equil}}^{(\text{nano})} \approx 0.35\,, \qquad (31)$$

that is significantly smaller than $\alpha_{\text{asympt}}^{(\text{nano})}$. We believe that such a large difference could be detected, even though the appropriate experimental setup might be difficult to realize. In particular in setup A the two nanotubes should approach each other at a speed comparable to (or even higher than) the Fermi velocity ($\sim 10^6$m/s). We note, by the way, that this problem does not arise in cold atomic systems, where the Fermi velocity is orders of magnitude smaller.

In conclusion we have computed the time-dependent evolution of the single-particle correlation function of two Luttinger liquids coupled by a sudden switch-on of the inter liquid interaction. This allows to evaluate the critical exponent α governing some physical observables accessible to real experiments, such as the momentum distribution function. We find that the in the long-time limit the initial ground state relaxes to a stationary state which is not the equilibrium ground state of the coupled system. In the latter case the critical exponent α_{equil} results to be smaller than the asymptotic exponent α_{asympt} of about a factor 2. An experiment capable to detect such a remarkable finding in carbon nanotubes is proposed. Finally we believe that the present study could be also relevant in experiments involving ultracold fermionic atoms loaded in optical lattices, where tunable Luttinger liquids might be realized in the near future.

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